

# RESERVATION STRATEGIES FOR STOCHASTIC JOBS

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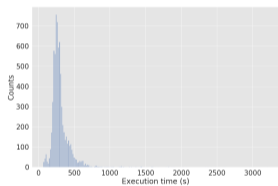
1. Inria & Univ Bordeaux;
2. Vanderbilt University;
3. ENS Lyon & UTK

Datamove & Polaris Seminar, december 2018

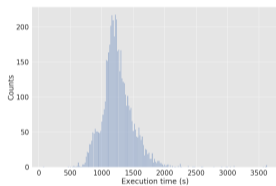


# MOTIV: NEUROSCIENCE APPLICATIONS

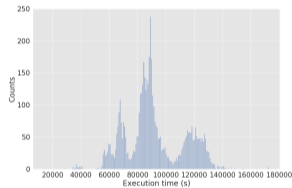
Often the execution time of an application is unknown before it runs.



(a) fMRIQA



(b) VBMQA



(c) dtiQA

Figure: Traces [2013-2016] of neuroscience apps (Vanderbilt's medical imaging database).

These applications are input dependent, but predicting the exact execution time is hard even when knowing the input.

## MOTIV: COMPUTING IN THE CLOUD

Several cost models to compute in the cloud:

- ▶ **On-Demand (OD):** “you pay for compute capacity by per hour or per second depending on which instances you run” (Amazon AWS).

(= **Pay what you use**)

- ▶ **Reserved-Instances (RI):** “Reserved Instances provide you with a significant discount (up to 75%) compared to On-Demand instance pricing.”

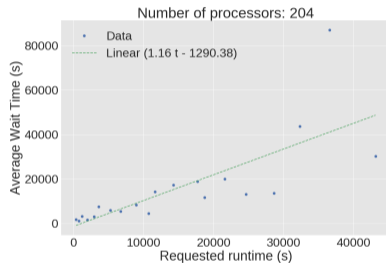
(= **Pay what you ask for**)

Payment Option	Upfront	Monthly*	Effective Hourly**	Savings over On-Demand	On-Demand Hourly
No Upfront	\$0.00	\$8.03	<u>\$0.011</u>	57%	\$0.0255
Partial Upfront	\$134.00	\$3.72	<u>\$0.01</u>	60%	
All Upfront	\$252.00	\$0.00	<u>\$0.01</u>	62%	

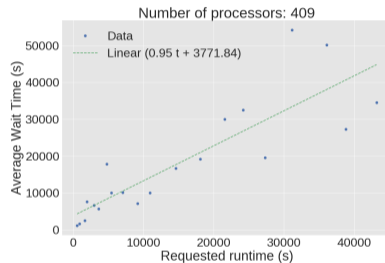
Figure: Data extracted from AWS website, 12/10/2018.

# MOTIV: COMPUTING IN HPC

Execution time = Wait time + Runtime:



(a) Jobs that requested 204 procs.



(b) Jobs that requested 409 procs.

Figure: Average wait times of jobs run on Intrepid (2009) as a function of requested runtime (data: Parallel Workload Archive).

**Observation:** For a given number of processors, wait time is an increasing function of requested time.



- ▶ Job execution time follows a Random Variable  $X$ .
  - ▶ Distribution  $\mathcal{D}$
  - ▶ Cumulative function (CDF)  $F$  ( $F(x) = \mathbb{P}(X \leq x)$ )\*
  - ▶ Density function (PDF)  $f$
  - ▶ Support is positive ( $X \in [\min_{\mathcal{D}}, \max_{\mathcal{D}}]$ , s.t.  $\min_{\mathcal{D}} \geq 0$  and  $\max_{\mathcal{D}} \in \mathbb{R} \cup \{\infty\}$ )
  
- ▶ Deterministic jobs (two executions of the same job have the same duration).

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\*most of the results assume a smooth CDF . . . . .

## PLATFORM COST MODEL

We consider a platform where one can request time:

- ▶ Assume a user **reserves** a set of resource for a time  $T$ ;
- ▶ The resources are **needed** for a time  $t$ .

$$\text{Cost: } \alpha T + \beta \min(T, t) + \gamma \quad (1)$$

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$\alpha, \beta, \gamma =$  platform parameters (e.g., OD:  $\alpha = \gamma = 0$ , RI:  $\beta = \gamma = 0$ )

- ▶ Reservation cost: what is paid for the reservation.
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## RESERVATION-BASED APPROACH

Given a job  $J$  of duration  $\mathbf{t}$  (unknown). The user makes a reservation of time  $t_1$ . Two cases:

- ▶  $\mathbf{t} \leq t_1$  The reservation is enough and the job succeeds.
- ▶  $\mathbf{t} > t_1$  The reservation is not enough. The job fails. The user needs to ask for another reservation  $t_2 > t_1$ .

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Let  $S = (t_1, t_2, \dots, t_{k-1}, t_k)$  be a sequence of reservations, s.t.  $t_1 < \dots < t_{k-1} < \mathbf{t} \leq t_k$ . The cost to the user is:

$$C(k, t) = \sum_{i=1}^{k-1} (\alpha t_i + \beta t_i + \gamma) + \alpha t_k + \beta t + \gamma$$

## OPTIMIZATION PROBLEM

Given a sequence of increasing reservation  $S = (t_1, t_2, \dots, t_i, t_{i+1}, \dots)$ .

Given a distribution of execution time  $X$  (PDF  $f$ ). The expected cost of  $S$  on  $X$  is:

$$\mathbb{E}(S) = \sum_{k=1}^{\infty} \int_{t_{k-1}}^{t_k} C(k, t) f(t) dt \quad (2)$$

### Definition (STOCHASTIC)

Given a stochastic job of execution time  $X$ . Given  $\alpha, \beta, \gamma$  defining a cost function (Eq. (1)), find a reservation sequence  $S$  with minimal expected cost  $\mathbb{E}(S)$  (Eq. (2)).

- ▶ (Almost) Characterization of optimal sequence for **any** distribution!
- ▶ RI strategy is in general useful, even for stochastic job!
- ▶ Comparison to “natural” greedy strategies show the importance of the optimal sequence.

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- ▶ Comparison to “natural” greedy strategies show the importance of the optimal sequence.

Additionally, opens up a whole venue of new fun scheduling problems 😊

As a preliminary lemma (warm up):

$$\begin{aligned}\mathbb{E}(S) &= \sum_{k=1}^{\infty} \int_{t_{k-1}}^{t_k} C(k, t) f(t) dt \\ &= \beta \cdot \mathbb{E}[X] + \sum_{i=0}^{\infty} (\alpha t_{i+1} + \beta t_i + \gamma) \mathbb{P}(X \geq t_i)\end{aligned}$$

Reminder:  $t_0 = 0$ ;  $C(k, t) = \sum_{i=1}^{k-1} (\alpha t_i + \beta t_i + \gamma) + \alpha t_k + \beta t + \gamma$



# EXISTENCE OF A SOLUTION

## Existence

Q1: Existence of a solution  $S = (t_1, t_2, \dots, t_i, t_{i+1}, \dots)$  with finite expected cost?

Yes†! Constructive proof:  $(t_i)_i = (\min_{\mathcal{D}} + i)_i$  has expectation  $\beta \cdot \mathbb{E}(X) + \alpha T_1 + \gamma$ , where

$$T_1 = \mathbb{E}[X] + 1 + \frac{\alpha + \beta}{2\alpha} (\mathbb{E}[X^2] - \min_{\mathcal{D}}^2) + \frac{\alpha + \beta + \gamma}{\alpha} (\mathbb{E}[X] - \min_{\mathcal{D}})$$

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†Conditions:  $\text{Var}[X] < \infty$

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Denote  $S^o = (t_1^o, t_2^o, \dots, t_i^o, \dots)$  an optimal sequence for STOCHASTIC. **Coro:**  $t_1^o \leq T_1$

---

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## Theorem (Optimal algorithm)

For  $X$  smooth, STOCHASTIC reduces to finding  $t_1^o$  that minimizes:

$$\sum_{i=0}^{\infty} (\alpha t_{i+1} + \beta t_i + \gamma) \mathbb{P}(X \geq t_i) \quad \text{s.t. for } i \geq 1 \quad F(t_i^o) = 1 \text{ or } t_{i+1}^o = g_{X,\alpha,\beta,\gamma}(t_i^o, t_{i-1}^o)$$

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  - ① For  $t = t_i^o$
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We are still missing  $t_1^o!$  (by convention we set  $t_0^o = 0$ )

## BRUTE-FORCE PROCEDURE FOR $t_1^o$

We know:  $t_1^o \in [\min_{\mathcal{D}}; T_1]$  ( $T_1$  = upper bound from earlier).

► Let  $S_{\text{bf}}(t_1) = (t_0, t_1, \dots, t_i, \dots)$  s.t.:

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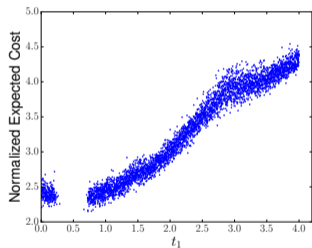
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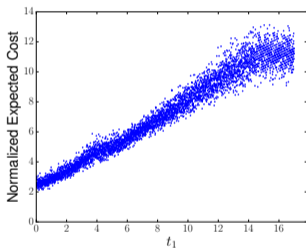
### **Brute-force procedure:**

- ▶  $t_1 = \min_{\mathcal{D}} + m \cdot \frac{T_1 - \min_{\mathcal{D}}}{M}, \forall m = 1, \dots, M$  (in practice we chose  $M = 5000$ ):
  - ▶ Expected cost of each sequence via a Monte-Carlo process
    - ▶ Randomly draw  $N$  execution time from distribution ( $N = 1000$  here)
    - ▶ Evaluate average cost over  $N$  samples of  $S_{\text{bf}}(t_1)$  ( $\mathcal{O}(MN)$ )
- ▶ Keep the best:  $t_1^{\text{algo}}$ .

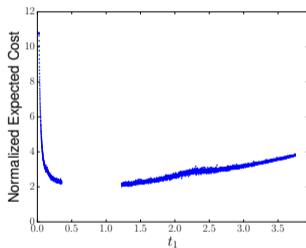
# IN PRACTICE, MONTE-CARLO SIMULATIONS



(a) Exponential Distribution



(b) Weibull Distribution

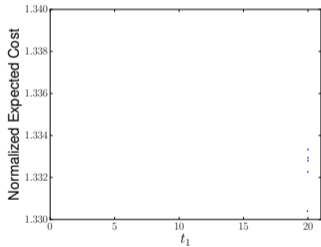


(c) Gamma Distribution

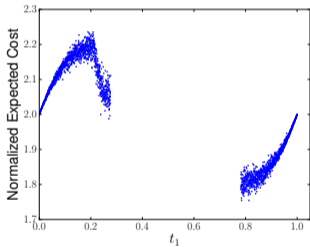
Figure: Monte-Carlo simulations, normalized =  $\text{Cost}/\mathbb{E}(X)$ ,  $\alpha = 1$ ,  $\beta = \gamma = 0$ .



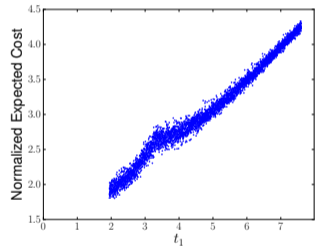
# IN PRACTICE, MONTE-CARLO SIMULATIONS



(a) Uniform Distribution



(b) Beta Distribution



(c) Bounded Pareto Distribution

Figure: Monte-Carlo simulations, normalized =  $\text{Cost}/\mathbb{E}(X)$ ,  $\alpha = 1$ ,  $\beta = \gamma = 0$ .

# DYNAMIC PROGRAMMING ALGORITHM

## Theorem

If  $X \sim (v_i, f_i)_{i=1\dots n}$  a discrete distribution, STOCHASTIC can be solved in polynomial time.

► Dynamic Programming algorithm:

$$\mathbb{E}_i^* = \min_{i \leq j \leq n} \left( \alpha v_j + \gamma + \sum_{k=i}^j f'_k \cdot \beta v_k + \left( \sum_{k=j+1}^n f'_k \right) (\beta v_j + \mathbb{E}_{j+1}^*) \right)$$

$$f'_k = \frac{f_k}{\sum_{j=i}^n f_j}, \forall k = i, \dots, n$$

$$\mathbb{E}_n^* = \alpha v_n + \beta v_n + \gamma$$

► Complexity:  $\mathcal{O}(n^2)$

# TRUNCATION AND DISCRETIZATION

To use the previous Theorem, we can discretize a continuous probability distribution.

- ▶ Truncation + Discretization: Given a precision  $\varepsilon$ ;
  - ▶ Change support to  $[\min_{\mathcal{D}}, Q(1 - \varepsilon)^{\ddagger}]$  (for infinite support).
  - ▶ Discretize the support:  $n$  discrete values:  $(v_i, f_i)_{i=1\dots n}$

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- ▶ Two discretization schemes
  - EQUAL-PROBABILITY: all discrete chunks have same probability
  - EQUAL-TIME: all chunks equally spaced in  $[\min_{\mathcal{D}}, Q(1 - \varepsilon)]$

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- ▶ Evaluation:  $\varepsilon = 10^{-7}$ ,  $n = 1000$

---

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▶  $\mu = \mathbb{E}(X) = \int_0^\infty tf(t)dt$      $\sigma^2 = \mathbb{E}(X^2) - \mu^2$

▶ 4 different heuristics

▶ MEAN-BY-MEAN:

$$t_i = \mathbb{E}(X|X > t_{i-1}) = \frac{\int_{t_{i-1}}^\infty tf(t)dt}{1 - F(t_{i-1})}, \quad \forall i \geq 2$$

▶ MEAN-DOUBLING:

$$t_i = 2^{i-1}\mu, \quad \forall i \geq 2$$

▶ MEAN-STDEV:

$$t_i = \mu + (i-1)\sigma, \quad \forall i \geq 2$$

▶ MEDIAN-BY-MEDIAN:

$$t_i = Q\left(1 - \frac{1}{2^i}\right), \quad \forall i \geq 2$$

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- ▶ Evaluation over two reservation-based scenarios:
  - ▶ RESERVATIONONLY: “pay what you request”:  $\alpha = 1, \beta = \gamma = 0$
  - ▶ NEUROHPC:
    - ① waiting time:  $\beta = 1, (\alpha, \gamma)$  by curve fitting waiting time from platform data
    - ② execution time: neuroscience application fitting

# RESERVATION ONLY: GENERAL HEURISTICS PERFORMANCE

Distribution	BRUTE-FORCE	MEAN-BY-MEAN	MEAN-STDEV	MEAN-DOUB.	MED-BY-MED	EQUAL-TIME	EQUAL-PROB.
<b>Gamma</b>							
<b>Uniform</b>							
<b>TruncatedNormal</b>							
<b>Beta</b>							

Table: Normalized expected costs. Value in brackets = expected costs normalized by BRUTE-FORCE performance.

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<b>Gamma</b>	2.02	2.45 (1.21)	2.26 (1.12)	2.22 (1.10)	2.66 (1.31)	2.14 (1.06)	2.09 (1.03)
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<b>Uniform</b>	1.33	2.21 (1.66)	1.86 (1.40)	1.69 (1.27)	2.22 (1.67)	1.33 (1.00)	1.33 (1.00)
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<b>Uniform</b>	1.33	2.21 (1.66)	1.86 (1.40)	1.69 (1.27)	2.22 (1.67)	1.33 (1.00)	1.33 (1.00)
<b>TruncatedNormal</b>	1.34	1.96 (1.46)	1.83 (1.36)	2.02 (1.50)	2.17 (1.61)	1.36 (1.01)	1.38 (1.03)
<b>Beta</b>	1.75	2.06 (1.18)	2.09 (1.19)	1.93 (1.10)	2.48 (1.42)	1.80 (1.03)	1.77 (1.01)

Table: Normalized expected costs. Value in brackets = expected costs normalized by BRUTE-FORCE performance.



# RESERVATION ONLY: RI OR OD?

*Reserved-Instance* better than *On-demand*:

$$c_{RI} \cdot \tilde{\mathbb{E}}(S) \leq c_{OD} \cdot \mathbb{E}(X)$$

$$\frac{\tilde{\mathbb{E}}(S)}{\mathbb{E}(X)} \leq \frac{c_{OD}}{c_{RI}}$$

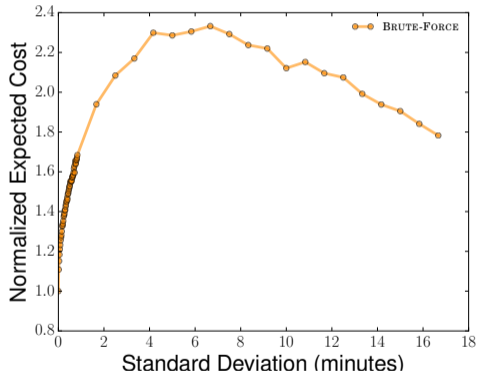


Figure: Truncated Normal distribution when  $\sigma$  varies from 0 to  $2\mu$  ( $\mu \approx 8\text{min}$ ).

# RESERVATIONONLY: DISCRETIZATION PERFORMANCE

Distribution	EQUAL-TIME							EQUAL-PROBABILITY						
	$n = 10$	$n = 25$	$n = 50$	$n = 100$	$n = 250$	$n = 500$	$n = 1000$	$n = 10$	$n = 25$	$n = 50$	$n = 100$	$n = 250$	$n = 500$	1000
Exponential														
Lognormal														
Weibull														
Pareto														
Uniform														

Table: Normalized expected costs of the two discretization-based heuristics with different numbers of samples in the RESERVATIONONLY scenario.

# RESERVATIONONLY: DISCRETIZATION PERFORMANCE

Distribution	EQUAL-TIME						EQUAL-PROBABILITY							
	$n = 10$	$n = 25$	$n = 50$	$n = 100$	$n = 250$	$n = 500$	$n = 1000$	$n = 10$	$n = 25$	$n = 50$	$n = 100$	$n = 250$	$n = 500$	1000
<b>Exponential</b>	2.64	2.32	2.43	2.49	2.28	2.39	2.33	3.66	2.88	2.35	2.41	2.35	2.32	2.43
<b>Lognormal</b>	2.02	1.92	1.97	1.93	1.90	1.93	1.89	2.99	2.32	2.13	1.99	1.87	1.93	1.99
<b>Weibull</b>														
<b>Pareto</b>														
<b>Uniform</b>														

Table: Normalized expected costs of the two discretization-based heuristics with different numbers of samples in the RESERVATIONONLY scenario.

# RESERVATIONONLY: DISCRETIZATION PERFORMANCE

Distribution	EQUAL-TIME							EQUAL-PROBABILITY						
	$n = 10$	$n = 25$	$n = 50$	$n = 100$	$n = 250$	$n = 500$	$n = 1000$	$n = 10$	$n = 25$	$n = 50$	$n = 100$	$n = 250$	$n = 500$	1000
<b>Exponential</b>	2.64	2.32	2.43	2.49	2.28	2.39	2.33	3.66	2.88	2.35	2.41	2.35	2.32	2.43
<b>Lognormal</b>	2.02	1.92	1.97	1.93	1.90	1.93	1.89	2.99	2.32	2.13	1.99	1.87	1.93	1.99
<b>Weibull</b>	17.00	7.15	4.45	3.33	2.49	2.56	2.44	18.69	9.03	5.14	3.60	2.88	2.47	2.57
<b>Pareto</b>	31.54	13.02	6.84	3.79	2.12	1.75	1.74	35.49	11.73	9.99	5.97	2.89	2.59	1.78
<b>Uniform</b>														

Table: Normalized expected costs of the two discretization-based heuristics with different numbers of samples in the RESERVATIONONLY scenario.

# RESERVATIONONLY: DISCRETIZATION PERFORMANCE

Distribution	EQUAL-TIME							EQUAL-PROBABILITY						
	$n = 10$	$n = 25$	$n = 50$	$n = 100$	$n = 250$	$n = 500$	$n = 1000$	$n = 10$	$n = 25$	$n = 50$	$n = 100$	$n = 250$	$n = 500$	1000
<b>Exponential</b>	2.64	2.32	2.43	2.49	2.28	2.39	2.33	3.66	2.88	2.35	2.41	2.35	2.32	2.43
<b>Lognormal</b>	2.02	1.92	1.97	1.93	1.90	1.93	1.89	2.99	2.32	2.13	1.99	1.87	1.93	1.99
<b>Weibull</b>	17.00	7.15	4.45	3.33	2.49	2.56	2.44	18.69	9.03	5.14	3.60	2.88	2.47	2.57
<b>Pareto</b>	31.54	13.02	6.84	3.79	2.12	1.75	1.74	35.49	11.73	9.99	5.97	2.89	2.59	1.78
<b>Uniform</b>	1.33	1.33	1.33	1.33	1.33	1.33	1.33	1.33	1.33	1.33	1.33	1.33	1.33	1.33

Table: Normalized expected costs of the two discretization-based heuristics with different numbers of samples in the RESERVATIONONLY scenario.

# NEUROHPC SCENARIO RESULTS

Instantiation from Neuroscience app (LogNormal distrib) + Intrepid waiting time cost.

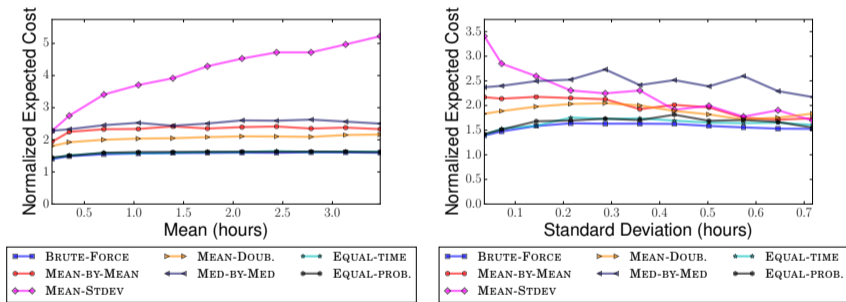


Figure: Impact of the mean or standard deviation in NEUROHPC scenario ( $\mu = 7.1128, \sigma = 0.2039$ ) with  $\alpha = 0.95, \beta = 1.0, \gamma = 1.05$ .

- ▶ Contributions
  - ▶ Existence of optimal reservation sequence
  - ▶ Characterization up to duration of first reservation  $t_1^o$
  - ▶ Upper-bound on  $t_1^o$
  - ▶ Heuristics and comprehensive simulation results
  
- ▶ Future works
  - ▶ Requests with variable amount of resources (time + # processors)
  - ▶ Checkpoints at the end of some/all reservations
  - ▶ Trade-off
    - ▶ useful works under reservations
    - ▶ sacrifice of time to avoid losing all current reservation work

Thanks